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Passage of a Swept Airfoil through an Oblique Gust

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An analysis is presented which yields an approximate solution for the unsteady aerodynamic response of an infinite swept wing encountering a vertical oblique gust in a compressible stream. The approximate expressions are of closed form and do not require excessive computer storage or computation time, and further, they are in good agreement with the results of exact theory. This analysis is used to predict the unsteady aerodynamic response of a helicopter rotor blade encountering the trailing vortex from a previous blade. Significant effects of three dimensionality and compressibility are evident in the results obtained.

Nomenclature

a	= speed of sound
C	= cosine Fresnel integrals
E	= complex Fresnel integral, defined by Eq. (25)
H_0, H_1	= Hankel functions, first kind, orders 0 and 1
I_0, I_1	= modified Bessel functions, first kind, order 0 and 1
J_0, J_1	= Bessel functions, first kind, orders 0 and 1
$\bar{K}$	= gust wave number
M	= Mach number of the freestream, U/a
n	= integer
r	= radial distance from the midchord line of the airfoil
R	= dimensionless radial distance
S	= sine Fresnel integral
t	= time
$\overline{u_2}$	= complex amplitude of $u_2{}'$
x_i	= Cartesian coordinates, Fig. 1
X_i	= dimensionless coordinates, defined by Eqs. (9) and (10)

Subscript

1/4	= quarter-chord

Introduction

THE determination of the unsteady aerodynamic response of an airfoil to a vertical gust velocity field has long been of interest to aeroelasticians and acousticians. For the most part aeroelasticians have used the incompressible two-dimensional theories of Kussner[1] and Sears[2] to predict the unsteady response. In estimating the acoustic field generated by airfoil-gust interactions the acoustician typically uses the unsteady lift obtained from one of the previous theories to determine the strength of the acoustic dipole source which is then assumed to replace the airfoil in the flowfield (e.g., Curle in Ref. 3).

Typical of the many unsteady aerodynamic problems in which these incompressible theories have been used in the past are: 1) rotor blades passing through the wakes of stator blades in turbomachinery, 2) an airfoil interacting with a turbulent gust, and 3) helicopter rotor blades encountering the tip vortices from proceding blades. In each of these problems some uncertainty arises as to the applicability of two-dimensional incompressible aerodynamics. It is not surprising, therefore, that several analytical studies have appeared recently which treat the complexities of three dimensionality and compressibility in the unsteady problem. For example, Filotas[4] obtained a closed form solution for an oblique sinusoidal gust encountering an infinite wing; however, this work is limited to a two-dimensional airfoil of zero sweep (i.e., the incoming flow is normal to the leading edge line of the wing) in an incompressible stream. Graham[5] included the effects of compressibility and three dimensionality but neglected the effects of sweep. His analysis is based on a numerical solution of the governing differential equations. Adamczyk[6,7] also included the effects of compressibility and three dimensionality, but the form of the solution was expressed in terms of an infinite series of Mathieu functions which are difficult to evaluate analytically. Another recent analysis by Johnson[8] included the effects of compressibility and three dimensionality; however, his approach is tailored towards analyzing the response of an airfoil to a free rectilinear vortex. Hence, the researcher has had no simple compressible three-dimensional analogue of the Sears solution available to him and was required to resort to the more restricted theories of Filotas

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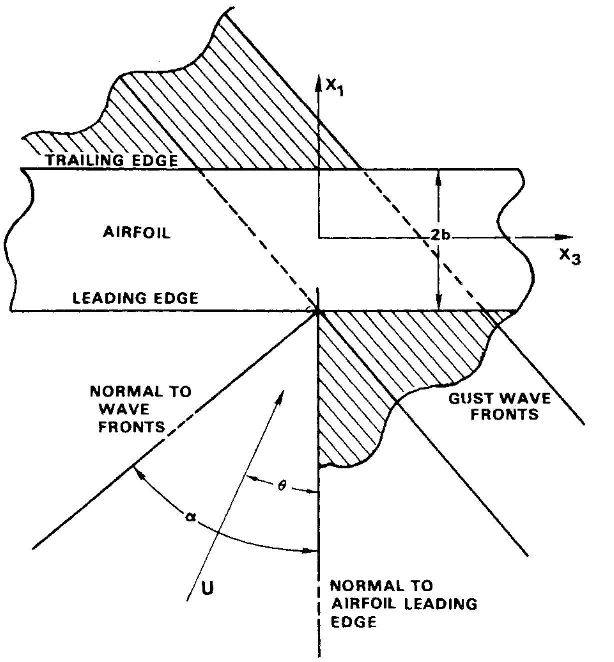

Fig. 1 Geometry of the encounter.

or Sears for predicting the aerodynamic response of an airfoil to a vertical gust.

To overcome the deficiencies described above, an analysis has been developed which generates simple approximate expressions for the unsteady compressible lift and moment response functions for an infinite swept airfoil encountering a three-dimensional oblique gust (see Fig. 1). The approximate expressions are of closed form and do not require excessive computer storage or computational time. This paper reports on the development of these approximate expressions, a comparison of results with an exact theory, and sample computations of the unsteady loading of a helicopter blade encountering a vortex.

Formulation of the Mathematical Problem

Governing Equation

Figure 1 shows the geometry of the problem. A infinite swept airfoil of chord 2b is held rigidly in a compressible fluid which is flowing over it with uniform velocities $U\cos\Theta$ in the x_1 direction, and $U\sin\Theta$ in the x_3 direction. Superposed on U is an unsteady three-dimensional disturbance u_i. The pressure and density are $p_0 + p$, and $\rho_0 + \rho$, respectively, where the subscripted part denotes the mean value, while the remaining part is associated with the disturbance.

The problem is described by the customary fluid dynamic equations for a compressible flow.[9] The equations of conservation of mass and momentum are linearized by assuming that the perturbations are small compared to the mean values thereby allowing the squares and products of the perturbations (and their derivatives) to be neglected while first-order terms are retained. In Ref. 11 it is shown that the linearized continuity and momentum equations can be combined by introducing a velocity potential to yield the governing differential equation for the problem.

$$a^2\left(\nabla^2\phi - M^2\cos^2\theta\frac{\partial^2\phi}{\partial x_1^2} - M^2\sin^2\theta\frac{\partial^2\phi}{\partial x_3^2}\right)$$

$$-2a^2\left(\frac{M\cos\theta}{a}\frac{\partial^2\phi}{\partial x_1\partial t} + \frac{M\sin\theta}{a}\frac{\partial^2\phi}{\partial x_3\partial t}\right.$$

$$\left. + M^2\cos\theta\sin\theta\frac{\partial^2\phi}{\partial x_1\partial x_3}\right) - \frac{\partial^2\phi}{\partial t^2} = 0 \qquad (1)$$

Here ϕ is the velocity potential. Equation (1) may be transformed to the Helmholtz equation (details can be found in Ref. 11)

$$(\partial^2\Phi/\partial X_1^2) + (\partial^2\Phi/\partial X_2^2) + \gamma^2\Phi = 0 \qquad (2)$$

for the modified velocity potential Φ where γ is a compressible cutoff parameter defined as

$$\gamma = \frac{\bar{K}b\sin\alpha}{1 - M^2\cos^2\theta}\left[\left(\frac{M\cos\theta}{\sin\alpha}\right)^2 - 1\right]^{1/2} = \frac{\bar{K}b\sin\alpha(M^{*2} - 1)^{1/2}}{1 - M^2\cos^2\theta} \qquad (3)$$

Note that the sign of γ^2 is dependent on whether the parameter $M^* = (M\cos\Theta/\sin\alpha)$ is greater than or less than one. This parameter is proportional to the phase velocity of the disturbance along the span of the airfoil relative to the oncoming flow. When M^* is greater than one, the relative phase velocity of the disturbance is supersonic. However, if M^* is less than one, the relative phase velocity is subsonic. The behavior of the solution of Eq. (2) at large distances from the airfoil is critically dependent on the value of M^*. This dependence can be shown by examining the asymptotic limit of the solution of Eq. (2) for an isolated acoustical dipole. This solution has the form

$$\Phi \sim \frac{e^{i\gamma R}}{(R)^{1/2}}\; R = (x_1^2 + x_2^2)^{1/2} \to \infty \qquad (4)$$

Equation (4) requires Φ to decay exponentially with distance for imaginary γ which, from Eq. (3), is equivalent to $M^* < 1$. If γ is real (i.e., $M^* \geq 1$), Eq. (4) is the asymptotic from for a cylindrical acoustical wave propagating outward from the origin at $R = 0$.

Boundary Conditions

A solution of Eq. (1) is sought which describes outgoing waves which must decay as $r^{-1/2}$ for large distances, r, from the airfoil. On the surface of the airfoil

$$(\partial\phi/\partial x_2) = -u_2{}^I \quad x_2 = 0 \quad -b \leq x_1 \leq b \qquad (5)$$

$u_2{}^I$ being a given function of space and time.

The present analysis assumes that the vertical velocity field induced by the incident disturbance on the airfoil is that of an oblique sinusoidal gust convected at the freestream velocity U. The mathematical form for this velocity field is

$$u_2{}^I = \bar{u}_2\exp\{i[\bar{K}\cos\alpha x_1 + \bar{K}\sin\alpha x_3 - \bar{K}U\cos(\theta - \alpha)t]\} \qquad (6)$$

The appropriate boundary conditions for the modified velocity potential Φ are derivable from Eqs. (4) and (5). These conditions are

$$\frac{\partial\Phi}{\partial X_2} = -\frac{b}{(1 - M^2\cos^2\theta)^{1/2}}\bar{u}_2 e^{i\beta X_1} \qquad (7)$$

$$\text{for } -1 \leq X_1 \leq 1, X_2 = 0$$

which was derived from Eq. (5) and

$$\Phi \sim R^{-1/2} \text{ as } R \to \infty \qquad (8)$$

which is the asymptotic radiation condition of Eq. (4). The frequency parameter β which appears in Eq. (7) is defined to be $\beta = \bar{K}b\cos\Theta/(1 - M^2\cos^2\Theta)$ while the dimensionless coordinates X_1, X_2 are defined as

$$X_1 = x_1/b \qquad (9)$$

$$X_2 = x_2(1 - M^2\cos^2\theta)^{1/2}/b \qquad (10)$$

In addition to the boundary conditions in Eq. (7) and Eq. (8) a Kutta-Joukowski condition must be imposed at the trailing edge of the airfoil. This condition requires that the pressure jump across the airfoil vanish at the trailing edge.

The governing differential equation (2) and the boundary conditions, Eq. (7) and Eq. (8) form a boundary value problem for Φ whose solution is dependent on only two variables, the cutoff parameter γ, and the frequency parameter β. The construction of the approximate solutions to this boundary value problem was divided into three parts: 1) a solution valid for small positive values of γ, 2) a solution valid for negative values of γ^2, and 3) a solution valid for large values of $|\gamma|$, each of which will be discussed below.

Solution of the Boundary Problem

General Considerations

An important objective of this analysis is to determine the unsteady lift and moment coefficients (per unit span) resulting from the encounter of an infinite swept airfoil with a three dimensional oblique sinusoidal gust in a compressible stream. In general the lift and moment coefficients can be written in the form

$$C_L = \left\{\frac{2\pi \bar{u}_2 \cos\theta}{U(1-M^2\cos^2\theta)^{1/2}}\right\} T_L e^{i[\bar{K}\sin\alpha x_3 - U\bar{K}\cos(\theta-\alpha)t]} \quad (11)$$

$$C_M = \left\{\frac{\pi \bar{u}_2 \cos\theta}{2U(1-M^2\cos^2\theta)^{1/2}}\right\} T_M e^{i[\bar{K}\sin\alpha x_3 - U\bar{K}\cos(\theta-\alpha)t]} \quad (12)$$

where T_L and T_M are the lift and moment transfer functions, and the expressions within the brackets { } represent the quasi-steady lift and moment response. The functions T_L and T_M will now be derived for the various values of γ of interest here.

Solution for Small Values of $\gamma \geq 0$

The approximate solution of Eq. (2) for small positive values of γ was obtained by expanding the exact solution to this problem, taken from Ref. 7, in a power series of γ.

The details of the approximate solution can be found in Ref. 11 in which the final form for the pressure distribution on the surface of the airfoil is shown to be

$$p^s = -\frac{\rho_0 U\bar{u}_2\cos\theta}{(1-M^2\cos^2\theta)^{1/2}}\left(\frac{1-X_2}{1+X_1}\right)^{1/2}\left[\frac{J_0(\beta)-iZJ_1(\beta)}{1+Z}\right] \exp\left[i\left[-\beta M^2\cos^2\theta X_1 + \frac{\bar{K}b\sin\alpha}{(1-M^2\cos^2\theta)^{1/2}}X_3 - \bar{K}U\cos(\theta-\alpha)t\right]\right] \quad (13)$$

where the variable

$$Z = -\frac{iH_0^{(1)}(\beta) + i(\gamma/\beta)^2[H_0^{(1)}(\gamma/2) - H_0^{(1)}(\beta)]}{H_1^{(1)}(\beta)} \quad (14)$$

The equation for the lift and moment transfer functions T_L and T_M, respectively, can be obtained by integrating the zero and first order moments of the pressure distribution over the chord of the airfoil. The resulting expression for the lift transfer function T_L is

$$T_L = \frac{J_0(\beta)-iZJ_1(\beta)}{1+Z}[J_0(\beta M^2\cos^2\theta) + iJ_1(\beta M^2\cos^2\theta)] e^{ib\bar{K}\cos\alpha} \quad (15)$$

while that for the moment transfer function T_M about the quarter-chord is:

$$T_{M_{1/4}} = e^{ib\bar{K}\cos\alpha}\left[\frac{J_0(\beta)-iZJ_1(\beta)}{1+Z}\right][-J_0(\beta M^2\cos^2\theta + J_2(\beta M^2\cos^2\theta) - i2J_1(\beta M^2\cos^2\theta)] + T_L \quad (16)$$

Equations (13, 15, and 16) are used to determine the aerodynamic loading for small positive values of γ.

Solution for Values of $\gamma^2 \leq 0$

A solution for the aerodynamic response functions for the entire region of $\gamma^2 \leq 0$ was developed by a similarity transformation of the analysis developed by Filotas.[4] The details of the approximate solution can be found in Ref. 11 in which the final form of the pressure distribution is shown to be.

$$p^S = \frac{\rho_0 U\bar{u}_2\cos\theta}{(1-M^2\cos^2\theta)^{1/2}}\frac{H(\gamma,\beta)}{I_0(\gamma)+I_1(\gamma)}\left(\frac{1-X_1}{1+X_1}\right)^{1/2}\exp\left[-(\gamma+i\beta M^2\cos^2\theta)X_1 + \frac{ib\bar{K}\sin\alpha}{(1-M^2\cos^2\theta)^{1/2}}X_3 - iU\bar{K}\cos(\theta-\alpha)t\right] \quad (17)$$

$$-1 \leq X_1 \leq 1$$

where

$$H(\gamma,\beta) = \exp\left\{-i(\gamma^2+\beta^2)\left[\cos\delta - \pi(\pi/2-\delta)\left(1+\frac{\sin\delta}{2}\right)\Big/\left(1+2\pi(\gamma^2+\beta^2)^{1/2}\left(1+\frac{\sin\delta}{2}\right)\right)\right]\right\} \Big/ \{1+\pi(\gamma^2+\beta^2)^{1/2}[1+\cos^2\delta+\pi(\gamma^2+\beta^2)^{1/2}\sin\delta]\} \quad (18)$$

and

$$\delta = \tan^{-1}\gamma/\beta \quad (19)$$

The lift transfer function T_L is computed by integrating the pressure distribution, Eq. (17), over the chord of the airfoil, yielding

$$T_L = e^{i\bar{K}b\cos\theta}\frac{I_0(\gamma+i\beta M^2\cos^2\theta) + I_1(\gamma+i\beta M^2\cos^2\theta)}{I_0(\gamma)+I_1(\gamma)}H(\gamma,\beta) \quad (20)$$

The corresponding expression for the moment transfer function T_M about the quarter-chord is

$$T_{M_{1/4}} = -\frac{2e^{i\bar{K}b\cos\alpha}}{I_0(\gamma)+I_1(\gamma)}\left[I_0(\gamma+i\beta M^2\cos^2\theta) + I_1(\gamma+i\beta M^2\cos^2\theta)\left(1-\frac{1}{\gamma+i\beta M^2\cos^2\theta}\right)\right]H(\gamma,\beta) + T_L \quad (21)$$

Equations (17, 20, and 21) can be used to compute the aerodynamic response functions over the entire range of negative γ^2.

Solution for Large Values of $|\gamma|^2$

The results predicted by the analysis of Ref. 7 for the pressure distribution on the surface of an airfoil encountering a two-dimensional sinusoidal gust indicate that for large values of $|\gamma^2|$ the pressure distribution appears to be composed of acoustical waves whose points of origin are the leading and trailing edges of the airfoil. This behavior implies that for large values of $|\gamma^2|$ a good approximation to the pressure field in the neighborhood of the leading edge (i.e., far upstream of the trailing edge) can be obtained by considering the interaction of a gust with a semiinfinite plate. For values of $\gamma^2 \neq 0$ the pressure distribution $p^{(1)}$ on the surface of the plate possesses a leading edge singularity, and decays monotonically with distance aft of the leading edge. Thus, the pressure distribution $p^{(1)}$ does not satisfy the Kutta-Joukowski condition. This error was corrected by adding to $p^{(1)}$ a pressure field $p^{(2)}$ which is equal and opposite to $p^{(1)}$ in the region $X_1 \geq 1$, $X_2 = 0$ and has its normal gradient vanishing vanishing everywhere in the region $X_1 \leq 1$, $X_2 = 0$.

The details of the approximate solution for large values

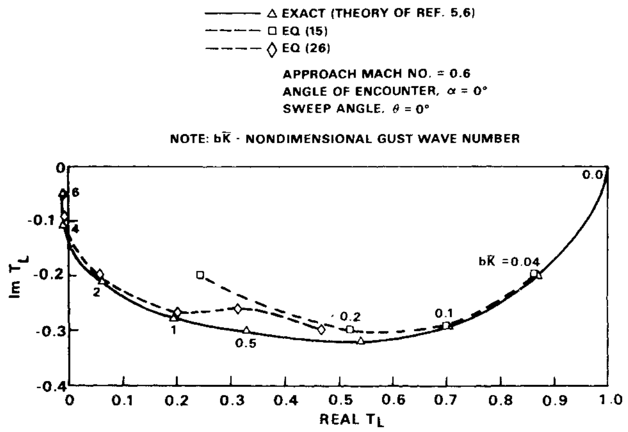

Fig. 2 A comparison between exact and approximate expressions for the lift transfer function for $\gamma \geq 0$.

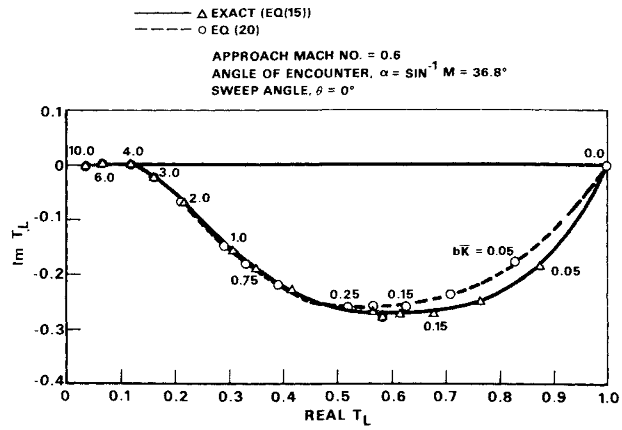

Fig. 3 A comparison between exact and approximate expressions for the lift transfer function for $\gamma = 0$.

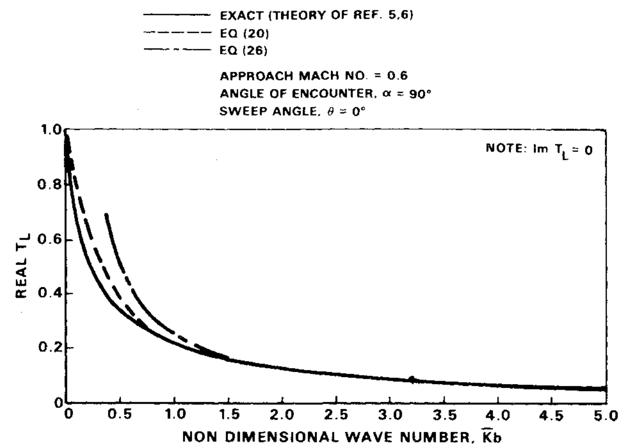

Fig. 4 A comparison between exact and approximate expressions for the lift transfer function for $\gamma \leq 0$.

of γ^2 can be found in Ref. 11 in which the final form for the pressure distribution on the surface of the airfoil is shown to be

$$p^S = p^{(1)} + p^{(2)} \tag{22}$$

where

$$p^{(1)} = \left\{ \frac{\rho_0 U \bar{u}_2 \cos\theta}{(1 - M^2 \cos^2\theta)^{1/2}} \frac{(1+i)}{(2X_1'\pi(\beta+\gamma))^{1/2}} \exp \right.$$
$$\left. [i(\gamma - \beta M^2 \cos^2\theta) X_1' + i\bar{K} \sin\alpha X_3 - iU\bar{K}\cos(\theta - \alpha)t] \right\} \tag{23}$$

and

$$p^{(2)} = \left\{ \frac{\rho_0 U \bar{u}_2 \cos\theta}{(1 - M^2 \cos^2\theta)^{1/2}} \left[\frac{\dfrac{1+i}{2} - E((2\gamma(2 - X_1'))^{1/2}}{(\pi(\beta+\gamma))^{1/2}} \right] \exp \right.$$
$$\left. [i(\gamma - \beta M^2 \cos^2\theta) X_1' + i\bar{K} \sin\alpha X_3 - iU\bar{K}\cos(\theta - \alpha)t] \right\}$$

$$X_1' = X_1 + 1, \; -1 \leq X_1 \leq 1 \tag{24}$$

where the complex Fresnel integral E is defined as

$$E(Z) = C(Z) + iS(Z) \tag{25}$$

Once again the lift and moment transfer function are obtained by integrating the zero and first order moment of the pressure distribution, Eq. (22), over the chord of the airfoil. The result for the lift transfer function T_L is

$$T_L = \frac{1}{[\pi(\beta+\gamma)]^{1/2}} \left\{ (1+i)\left(\frac{\pi}{\gamma - \beta M^2 \cos^2\theta}\right)^{1/2} \times \right.$$
$$E[(2(\gamma - \beta M^2 \cos^2\theta))^{1/2}] + [1 - e^{2i(\gamma - \beta M^2 \cos^2\theta)}]$$
$$\frac{1-i}{2(\gamma - \beta M^2 \cos^2\theta)} + \frac{iE[(4\gamma)^{1/2}]}{(\gamma - \beta M^2 \cos^2\theta)} - \frac{i(2\gamma)^{1/2}}{\gamma - \beta M^2 \cos^2\theta} \times$$
$$\left. \frac{e^{2i(\gamma - \beta M^2 \cos^2\theta)}}{(\gamma + \beta M^2 \cos^2\theta)^{1/2}} E[(2(\gamma + \beta M^2 \cos^2\theta))]^{1/2} \right\} \tag{26}$$

while that for the moment transfer function about the quarter-chord is

$$T_{M_{1/4}} = -\frac{(1-i)(\pi)^{1/2}}{(\gamma - \beta M^2 \cos^2\theta)^{3/2}} E\left[\left(\frac{4}{\pi}(\gamma - \beta M^2 \cos^2\theta)\right)^{1/2}\right] +$$
$$\frac{1+i}{(\gamma - \beta M^2 \cos^2\theta)^2}[1 - e^{2i(\gamma - \beta M^2 \cos^2\theta)}] - \frac{2E[2(\gamma)^{1/2}]}{(\gamma - \beta M^2 \cos^2\theta)^2} -$$
$$\frac{i2(2\gamma)^{1/2}}{(\gamma - \beta M^2 \cos^2\theta)} \frac{e^{2i(\gamma - \beta M^2 \cos^2\theta)}}{(\gamma + \beta M^2 \cos^2\theta)^{1/2}} \times$$
$$E[(2(\gamma + \beta M^2 \cos^2\theta))^{1/2}] \times$$
$$\left[2 - \frac{2}{2(\gamma + \beta M^2 \cos^2\theta)}\right] + 2\left(\frac{2\gamma}{\pi}\right)^{1/2} \frac{e^{4i\gamma}}{\gamma^2 - \beta^2 M^4 \cos^4\theta} +$$
$$\frac{2(2\gamma)^{1/2} e^{2i(\gamma - \beta M^2 \cos^2\theta)}}{(\gamma - \beta M^2 \cos^2\theta)^2 (\gamma + \beta M^2 \cos^2\theta)^{1/2}} E[2(\gamma +$$
$$\beta M^2 \cos^2\theta)^{1/2}] - T_L \tag{27}$$

Equations (22, 26, and 27) can be used to determine the aerodynamic response function for large values of $|\gamma|^2$.

Discussion of the Approximate Solutions

The objective of this section is to demonstrate the validity of the approximate solutions that were developed in the previous sections for the aerodynamic response functions. First, consider the equations for the lift transfer function, T_L, for both small and large positive values of γ (i.e., Eqs. (15) and (26), respectively), and compare the two dimensional limits of these equations to the exact results of Refs. 5 and 6. This comparison is presented in Fig. 2 in which the imaginary part of T_L is plotted as a function of the real part of T_L at specific values of nondimensional wave number, $b\bar{K}$, for a Mach number of 0.6. The solid curve appearing in this figure corresponds to the exact results presented in Refs. 5 and 6, while the dashed curves represent values computed from Eqs. (15) and (26). These results show that the present approximate solutions for the lift transfer function are in good agreement with the exact results over a wide range of $b\bar{K}$. The values of the lift transfer function for the region between the upper limit of Eq. (15) and the lower limit of Eq. (26) can be determined with sufficient accuracy for a linear curve fit which joins the limits of the two solutions. The accuracy of Eqs. (21) and (27) in predicting the moment transfer function is shown in Ref. 10 to be comparable to that attained for the lift transfer function.

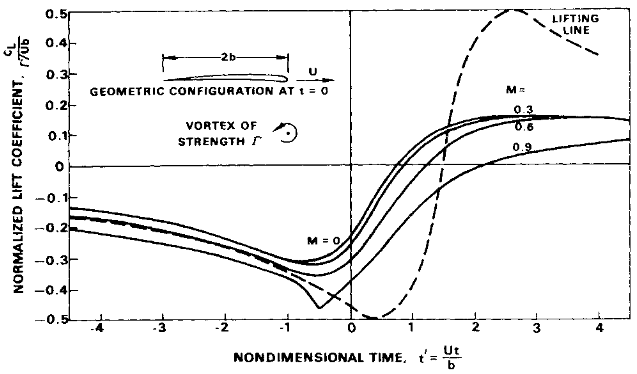

Fig. 5 Lift coefficient per unit span as a function of time and Mach number for a parallel encounter.

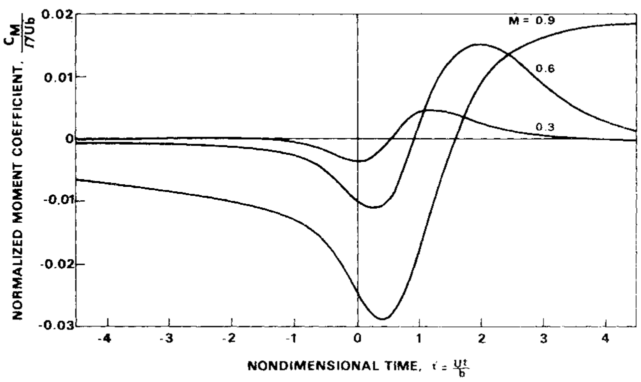

Fig. 6 Moment coefficient per unit span about the quarter-chord as a function of time and Mach number for a parallel encounter.

The accuracy of the approximate expression for the lift transfer function governing the region $\gamma^2 \leq 0$ is demonstrated by first comparing the results predicted by Eq. (20) with those predicted by Eq. (15) for $\gamma^2 = 0$. In Eq. (15) the limit as $\gamma \to 0^+$ yields exact results while the limit of Eq. (20) is approximate. A comparison for $\gamma^2 = 0$ is shown in Fig. 3 in which the imaginary part of T_L is plotted as a function of the real part of T_L at specific values of nondimensional wave number, bK, for a Mach number of 0.6. Once again the agreement between the exact solution and the approximate solution is good. To establish the accuracy of Eq. (20) for values of $\gamma^2 < 0$ a comparison was made with the results obtained from an analysis based on the theory of Ref. 7. This comparison is presented for the real part of T_L in Fig. 4, for an encounter angle of 90°, Mach number 0.6 and zero angle of sweep. (The imaginary part of T_L predicted by Eq. (20) is zero, which is the correct solution for this particular encounter.) The solid curve in Fig. 4 corresponds to the exact result while the dashed curve is that predicted by Eq. (20). In addition the result predicted by Eq. (26) is included as the dot-dash curve. The agreement between both approximations and the exact solution is very good for values of $b\bar{K} > 1.0$. For values of $b\bar{K} < 1.0$ the agreement between Eq. (20) and the exact result has deteriorated somewhat but is more than adequate for computational purposes. The comparisons for the moment transfer function T_M is presented in Ref. 10 and the agreement is shown to be comparable to that attained for the lift transfer function.

The validity of these approximate solutions is clearly demonstrated by the good agreement with the exact theory, and the closed form structure of the solutions facilitates their use. The latter is particularly important when these results are applied to specific problems of unsteady aerodynamic response where computer time and core storage may be limiting factors.

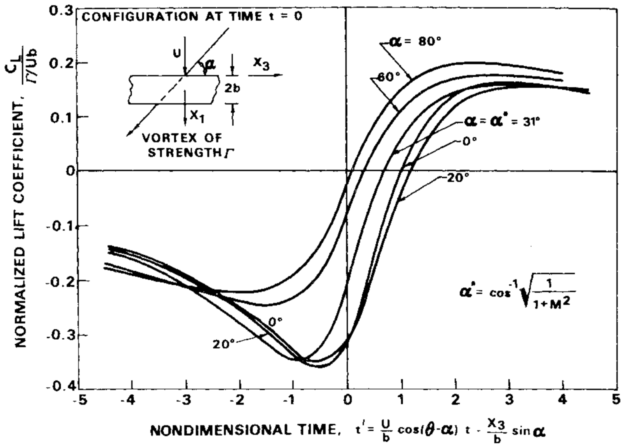

Fig. 7 Lift coefficient per unit span as a function of time for an oblique encounter.

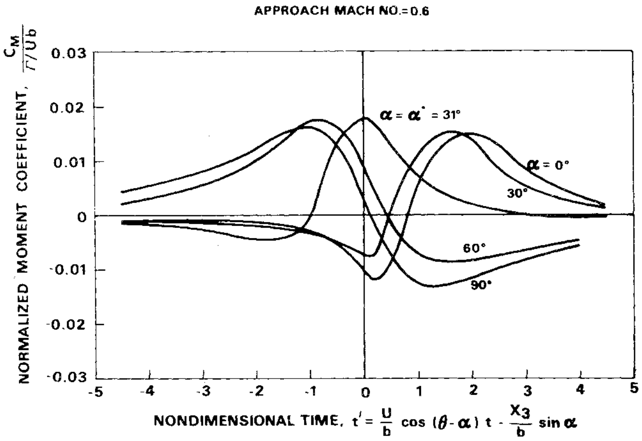

Fig. 8 Moment coefficient per unit span about the quarter-chord as a function of time for an oblique encounter.

Application of the Approximate Solutions

The theory of the preceding sections has been applied to the study of the lift and moment response of a helicopter rotor blade encountering the trailing vortex from a previous blade The model that was developed to simulate this aerodynamic interaction problem assumed the rotor blade to be an infinite swept airfoil. The trailing vortex was modeled as a rectilinear vortex translating at a velocity equal to the mean stream velocity.

The lift and moment coefficients per unit span generated by the encounter were obtained by the Fourier superposition of the lift and moment responses to an infinite series of oblique gusts which constituted the Fourier decomposition of the rectilinear vortex (details given in Ref. 10). A series of computations were performed to determine the effects of encounter angle, Mach number, and vertical vortex displacement on the lift and moment response and these results are presented in Figs. 5-12. In all cases examined, the airfoil sweep angle was fixed at zero (i.e., the incoming flow was normal to the leading edge line of the wing). The lift and moment coefficients per unit span appearing in these figures are normalized with respect to the parameter Γ/Ub where Γ is the circulation of the free vortex, U is the freestream velocity, and b is the semichord of the airfoil. A typical range of this parameter for a helicopter blade is $0.2 \leq \Gamma/Ub \leq 2$. Figure 5 shows the lift coefficient per unit span generated by a parallel encounter of a vortex with a two-dimensional wing as a function of freestream Mach number and nondimensional time. At $t' = 0$ the position of the vortex is one semichord below the airfoil leading edge, while at $t' = 2$ it is located one semi-

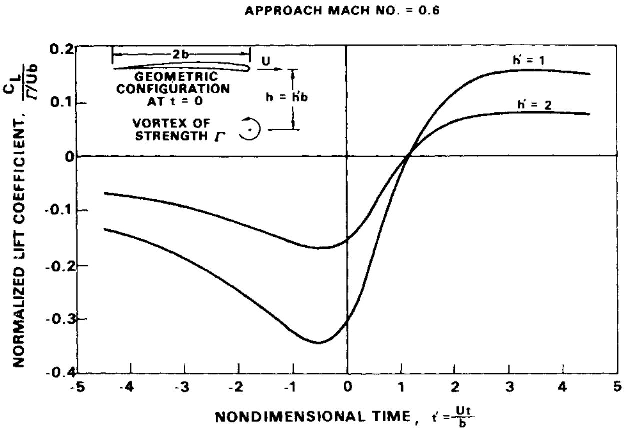

Fig. 9 Lift coefficient per unit span as a function of time and vortex vertical displacement for a parallel encounter.

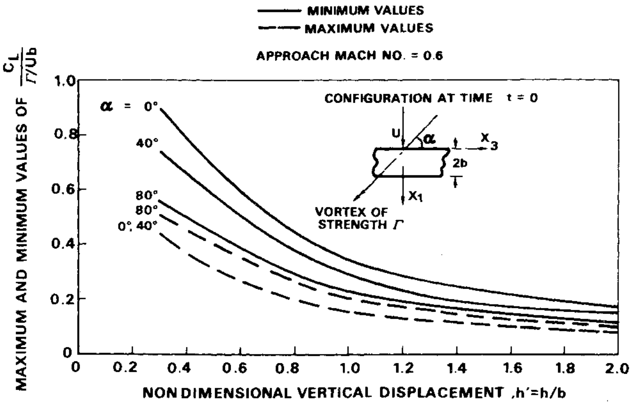

Fig. 11 Maximum and minimum values of the lift coefficient per unit span.

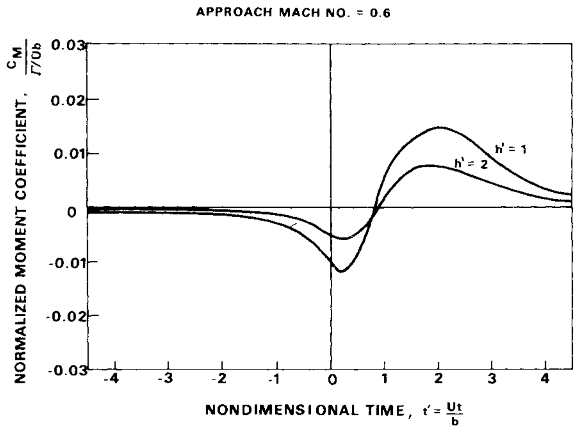

Fig. 10 Moment coefficient per unit span about the quarter-chord as a function of time and vortex vertical displacement for a parallel encounter.

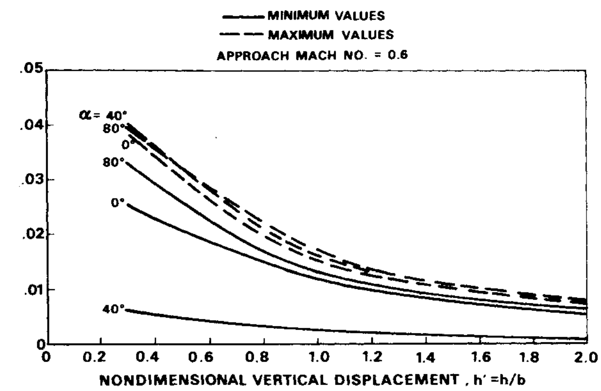

Fig. 12 Maximum and minimum values of the moment coefficient per unit span about the quarter-chord.

chord below the trailing edge. The lift coefficient calculated on the basis of quasi-steady lifting line theory is also shown as a dashed line in Fig. 5. This result is presented to illustrate the effects of unsteadiness. It is seen from the figure that lifting line theory overestimates the lift coefficient when the vortex is directly beneath ($0 \leq t' \leq 2$) or slightly downstream ($t' > 2$) of the airfoil. This behavior can be attributed to rapid time variations in downwash that occur during the time the vortex is beneath the airfoil. In Fig. 6 the moment coefficient about the quarter-chord produced by a parallel encounter is shown as a function of Mach number and nondimensional time t'. A nonzero moment coefficient about the quarter-chord for a parallel encounter can be directly attributed to compressibility, since the moment is zero for an incompressible fluid. (This is shown on page 287 of Ref. 9. In addition, it should be noted that lifting line theory always predicts zero pitching moment.) It is seen from Fig. 6 that significant values of $C_{M1/4}/(\Gamma/Ub)$ can be obtained, which increase with increasing Mach number.

The results appearing in Figs. 7 and 8 illustrate the effects of three dimensionality at a freestream approach Mach number of 0.6. These figures represent the lift and moment coefficients, respectively, produced by a vortex encountering an airfoil at an oblique angle as shown in the sketch in Fig. 7. Both results are presented as a function of nondimensional time t', where t' is more generally defined as

$$t' = \frac{Ut}{b} \cos(\theta - \alpha) - \frac{x_3}{b} \sin\alpha \tag{28}$$

In this equation α is the angle between the vortex and the leading edge of the wing, and x_3 is the spanwise coordinate. The spanwise position of the vortex at time $t = 0$ is $x_3 = 0$, while its vertical position is one semichord below the wing. The lift response shown in Fig. 7 appears to be a weak function of the encounter angle, while the corresponding moment response, shown in Fig. 8, appears to be very sensitive to variations in the encounter angle.

A number of computations were performed to determine the effect of the vertical position of the free vortex beneath the airfoil on the unsteady lift and moment response generated by an encounter. The results for the normalized lift and moment coefficients per unit span generated by a parallel encounter at a Mach number 0.6 are shown in Figs. 9 and 10 as functions of nondimensional time t' and nondimensional vertical distance $h' = h/b$ (where h is the vertical height of the free vortex beneath the wing). The geometry of the encounter at time $t = 0$ is as shown in the sketch of Fig. 9. These results indicate that the effect of increasing the vertical distance between the airfoil and the free vortex is to reduce the lift and moment response. Figures 11 and 12 show the maximum positive and negative values of the response curves as functions of vertical displacement and encounter angle. These results were obtained for an approach Mach number 0.6. Decreasing the vertical displacement between the free vortex and the wing causes the lift and moment response to increase for all encounter angles examined to date. These results also show that the maximum positive values of the lift and moment response curves appear to be relatively insensitive to the encounter angle. However, the maximum negative values are strongly dependent on the encounter angle.

Results and Conclusions

A primary objective in developing the present analysis was to significantly reduce the computational time required by the theory of Ref. 7 to predict the aerodynamic response resulting from the encounter of a two-dimensional swept airfoil with a three-dimensional oblique sinusoidal gust in a compressible stream. This general objective was achieved by deriving a simple set of approximate expressions for the unsteady pressure, lift and moment response functions. The validity of these expressions was demonstrated by the close agreement with the exact results (i.e., based on the theories of Refs. 5 and 7) over a wide range of values of the governing parameters.

The approximate expressions were used to predict the aerodynamic loading on a helicopter rotor blade encountering the tip vortex shed by a previous blade. The major conclusion reached in this application of the analysis is that compressibility, unsteadiness, and three-dimensionality are important terms which must be included to correctly predict the aerodynamic response, particularly if the vortex is located within two chords length of the airfoil.

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